53 (MA 101) ENMA-I

2016

ENGINEERING MATHEMATICS-I

Paper : MA 101

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Form differential equation of the family of curves $y = e^x (A\cos x + B\sin x)$, where A and B are arbitrary constants. 5

   (b) Examine the following series

   $$\frac{2}{3} + \frac{4}{3^2} + \frac{6}{3^3} + \frac{8}{3^4} + \cdots$$ 5

   (c) Find the nth derivative of the function $y = \cos^2 x \sin^3 x$ 5

   (d) Expand $\sin x$ in proper of $\left(x - \dfrac{\pi}{2}\right)$ and determine $\sin 91°$ correct to four decimal places. 5

2. (a) Discuss the convergency of the following

series $\sum\limits_{n} \dfrac{(n+1)^n x^n}{n^{n+1}}$ 6

(b) Trace the curve $a^2 y^2 = x^2 \left(a^2 - x^2\right)$. 6

(c) Solve : *(any two)* 4×2=8

(i) $(a + x)\dfrac{dy}{dx} = y - ay^2$

(ii) $y^2 + x^2 \dfrac{dy}{dx} = xy \dfrac{dy}{dx}$

(iii) $\left\{ y\left(1 + \dfrac{1}{x}\right) + \cos y \right\} dx + (x + \log x - x \sin y)\, dy = 0$

3. (a) If $y = e^{5 \sin^{-1} x}$, then show that

$\left(1 - x^2\right) y_{n+2} - (2n + 1)\, x y_{n+1} - \left(n^2 + 25\right) y_n = 0$ 5

(b) State limit comparison test. Is the series

$\sum\limits_{n} \left(\sqrt{n^5 + 1} - \sqrt{n^5 - 1} \right)$

convergent ? Justify. 1+4=5

(c) Find the co-ordinates of the point where the line joining the points $(2, -3, 1)$ and $(1, 2, -4)$ cuts the plane $2x + 3y - 5z + 3 = 0$. 5

(d) Find orthogonal trajectories of the family of curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter.   5

4. (a) Find the equation of the plane through the line of intersection of the planes $2x + 3y - 4z = 1$, $3x - y + z + 2 = 0$ and the origin.   4

   (b) Find the centre of curvature of the curve $y^2 = 4ax$ at (x, y).   4

   (c) Solve :   4×3=12

     (i) $\left(x + 2y^3\right)\dfrac{dy}{dx} = y$

     (ii) $\dfrac{dy}{dx} = e^{x-y}\left(e^x - e^y\right)$

     (iii) $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} = x^2 + 1$

5. (a) Show that the series $\sum\limits_{n}(-1)^n \sin\left(\dfrac{1}{n}\right)$ is not absolutely convergent.   5

   (b) Find the direction ratios of the line which is perpendicular to the lines with direction ratios (a, b, c) and (a', b', c').   3

(c) Find the asymptotes of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0 \qquad 5$$

(d) Evaluate :

$$\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta \, d\theta \qquad 3$$

(e) Find the equation of the sphere which passes through the points $(1, 2, 3)$, $(0, -2, 4)$, $(4, -4, 2)$ and $(3, 1, 4)$.   4

6. *(a)* Find the area enclosed by the loop of the curve $x^3 + y^3 = 3axy$.   5

(b) The curve $y^2(a + x) = x^2(3a - x)$ revolves about the axis of X. Find the volume generated by the loop.   5

(c) Solve : *(any two)*   5×2=10

   (i) $x^2 \dfrac{d^2y}{dx^2} - 4x \dfrac{dy}{dx} + 6y = x$

   (ii) $\left(D^2 + 6D + 9\right)y = 2e^{-3t}$, $D = \dfrac{d}{dt}$

   (iii) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = \cos 3x$