Total number of printed pages–8

53 (EC 603) DSPR

2015

DIGITAL SIGNAL PROCESSING

Paper : EC 603

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. (a) Consider a discrete time system characterized by the following input-output relation. Determine whether it is linear, memoryless, stable, causal and time invariant

$y(n) = x(n+2) - 2x(n-7) + 9$ 5

(b) An LTI system is characterized by an impulse response

$$h(n) = \left(\frac{3}{4}\right)^n u(n)$$

Find the step response of the system. Also evaluate output of the system at $n = \pm 5$. 5

Contd.

(c) Compute $y(n) = x(n) * h(n)$ where $x(n) = \alpha^n u(n)$ $h(n) = \beta^n u(n)$

$0 < \alpha,\ \beta < 1$

5

(d) Let $y(n)$ denote the convolution of $h(n)$ and $g(n)$ where $h(n) = \left(\frac{1}{2}\right)^n u(n)$ and $g(n)$ is causal sequence. If $y(0) = 1$ and $y(1) = \frac{1}{2}$, then find $g(1)$. 5

2. (a) Find the DTFT of the following signals

(i) $x(n) = a^{|n|}$ $|a| < 1$

(ii) $x(n) = u(n)$ 3+5=8

(b) Consider the filter

$$y(n) = 0 \cdot 9y(n-1) + bx(n)$$

Determine b so that $\left|H\left(e^{jo}\right)\right| = 1$ 2

(c) State and prove the time-shifting property of DTFT. 3

(d) Find the inverse DTFT of the rectangular pulse spectrum defined only for $-\pi \leq w \leq \pi$ 4

$$X\left(e^{jw}\right) = \begin{cases} 1 & |w| < w_c \\ 0 & w_c < |w| \leq \pi \end{cases}$$

(e) A discrete-time system with input $x(n)$ and output $y(n)$ is described by the relation

$$Y\left(e^{jw}\right)=e^{-jw}\times\left(e^{jw}\right)+\frac{d}{dw}\times\left(e^{jw}\right)$$

Find the output $y(n)$ if $x(n)=\delta(n)$

3

3. (a) Determine the z-transform of the following signal

$$x(n)=\frac{1}{3}\left(n^2+n\right)\left(\frac{1}{2}\right)^{n-1}u(n-1)$$ 4

(b) Determine the inverse z-transform

$$X(z)=\frac{1}{1-1\cdot 5z^{-1}+0\cdot 5z^{-2}}$$

if

(i) ROC $|z|>1$

(ii) ROC $|z|<0\cdot 5$

(iii) ROC $0\cdot 5<|z|<1$ 5

(c) When input $x(n)=u(n)+\left(-\frac{1}{2}\right)^n u(n)$ is applied to a linear causal time invariant system, the output is

$$y(n)=6\left(-\frac{1}{4}\right)^n u(n)-6\left(-\frac{1}{3}\right)^n u(n)$$

(i) Find the transfer function of the system.

(ii) What is the difference equation representation of the system?

4+3=7

(d) A causal LTI system is described by the difference equation

$$2y(n) = \alpha y(n-2) - 2x(n) + \beta x(n-1)$$

Find the value of α and β for which the system is stable. 4

4. (a) Let $x(n)$ be a real sequence defined by $x(n) = (1, 2, 3, 4)$. Without evaluating its DFT $X(K)$ find

(i) $\sum_{K=0}^{3} X(K)$

(ii) $X(0)$ 4

(b) Let $x(n)$ be real sequence of length N and its N-point DFT is given by $X(K)$ show that

(i) $X(N-K) = X*(K)$

(ii) $X(0)$ is real

(iii) if N is even, $X\left(\frac{N}{2}\right)$ is real 6

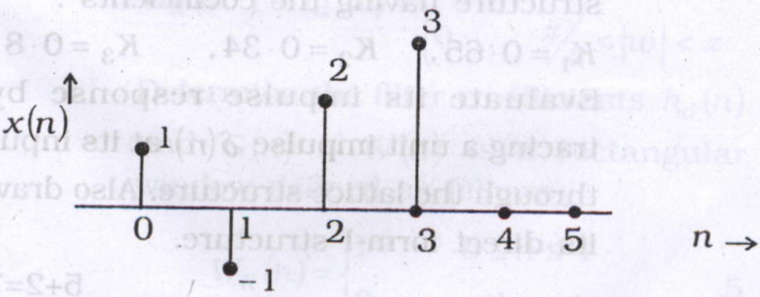
(c) Let $x(n)$ be a finite length sequence \ $X(K)=\{0, 1+j, 1, 1-j\}$. Using the properties of DFT, find the DFT of the following sequence

(i) $x_1(n)=e^{i\pi/2\, n}\; x(n)$

(ii) $x_2(n)=cos\left\{\left(\frac{\pi}{2}\right)n\right\} x(n)$

(iii) $x_3(n)=x\left((n-1)\right)_4$ 6

(d) Let $X(K)$ denote a 6- point DFT of a length 6 real sequence $x(n)$. The sequence is shown in the figure. Without computing the DFT find the length 6-sequence $y(n)$ whose 6-point DFT is given by $Y(K)=W_3^{2K}\; X(K)$. 4



5. (a) The transfer function of a discrete causal system is given as follows :

$$H(z)=\frac{1-z^{-1}}{1-0\cdot 2z^{-1}-0\cdot 15z^{-2}}$$

(i) Draw cascade and parallel realization.

(ii) Find the impulse response of the system. 5+2=7

(b) A linear time invariant system is described by the following input-output relation

$$2y(n)-y(n-2)-4y(n-3)=3x(n-2)$$

Realize the system using direct form-II realization. 6

(c) Consider a three-stage FIR lattice structure having the coefficients :

$K_1 = 0\cdot 65$, $K_2 = 0\cdot 34$, $K_3 = 0\cdot 8$

Evaluate its impulse response by tracing a unit impulse $\delta(n)$ at its input through the lattice structure. Also draw its direct form-I structure. 5+2=7

6. *(a)* Let $X(w)$ denote the Fourier transform of the sequence $x(n)=\left(\frac{1}{2}\right)^n u(n)$. Let $x_1(n)$ denote a sequence of finite duration of length 10; ie $x_1(n)=0$ for $n<0$ and $x_1(n)=0$ for $n\geq 10$. The 10 point DFT of $x_1(n)$ denoted by $x_1(K)$ corresponds to 10 equally spaced samples of $X(w)$; that is $X_1(K)=X(w)\big|_{w=\frac{2\pi k}{10}}$. Determine $x_1(n)$. 10

(b) Bring out the comparison between IIR and FIR filter. 5

(c) A lowpass filter is to be designed with the following desired frequency response

$$H_d\left(e^{iw}\right)=H_d(w)=\begin{cases} e^{-jzw} & , \quad |w| < \pi/4 \\ 0 & \pi/4 < |w| < \pi \end{cases}$$

Determine the filter co-efficients $h_d(n)$ and $G(n)$ if $W(n)$ is a rectangular window defined as follows

$$W_R(n)=\begin{cases} 1 & 0 \leq n \leq 4 \\ 0 & \text{otherwise} \end{cases}$$ 5

7. *(a)* Design a digital lowpass filter using Buterworth approximation by impulse invariance. The specifications are as follows

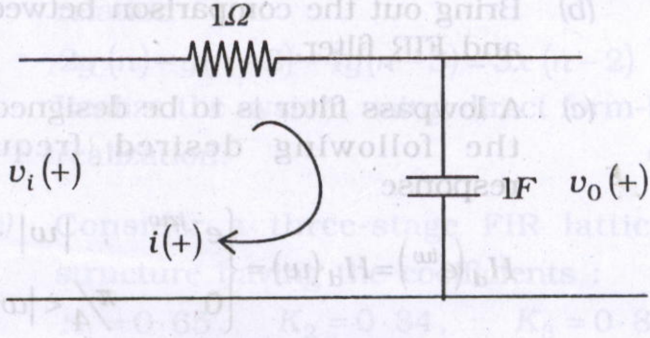
$0 \cdot 89125 \leq |H(\omega)| \leq 1$ for $0 \leq \omega \leq 0 \cdot 2\pi$

$|H(\omega)| \leq 0 \cdot 17783$ for $0 \cdot 3\pi \leq \omega \leq \pi$. 12

(b) Obtain the digital filter equivalent to analog filter shown in the following *fig.* using

(i) Impulse invariant transformation

(ii) Bilinear transformation



Assuming the sampling frequency $F_s = 8F_c$ where F_c is the cut-off frequency of the filter. 8

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