Total number of printed pages–7

53 (MA 401) NMCP

2015

NUMERICAL METHODS & COMPUTER PROGRAMMING

Paper : MA 401

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer Questions No. ***1*** *and* ***any four*** *from the rest.*

1. *(a)* Answer the following questions : 1×15=15

   (i) What do you mean by error ?

   (ii) If $\frac{2}{3}$ is approximated to 0·6667, find the absolute error.

   (iii) What is transcendental equation ?

   (iv) Under what condition Iteration method is convergent ?

Contd.

(v) What is the rate of convergence of Newton-Raphson method ?

(vi) What is the rate of convergence of Secant method ?

(vii) If $x = \sqrt{N}$, then $x_{n+1} = ?$

(viii) Define the principle of least square method.

(ix) Prove that $\Delta_{y,z}^{2}\left(x^3\right) = x + y + z$.

(x) $\Delta^2 e^x = ?$

(xi) What is the degree of Polynomial in terms of Simpson's $\frac{1}{3}rd$ rule ?

(xii) What do you mean by Numerical Integration ?

(xiii) Under what condition Simpson's $\frac{3}{8}$ th rule can be applied ?

(xiv) What do you mean by interpolation ?

(xv) In what sense Newton's Forward interpolation can be applied ?

(b) State True **or** False (T/F) : 1×5=5

(i) Euler's method for Numerical Solution of a differential equation is the best method.

(ii) Euler's Improve method is known as Heun's method.

(iii) Taylor's Series Solution for $\frac{dy}{dx} = f(x,y)$ is applicable only when the various partial derivatives of $f(x,y)$ exists.

(iv) The fourth order Runge-Kutta is the most extensively used method among the Runge-Kutta method.

(v) Predictor-corrector is a single step method.

2. (a) Write the geometrical Interpretation of Newton-Raphson method. 4

(b) By lagrange's formula, find the value of log 323·5 from the following data : 6

x :	321·0	322·8	324·2	325·0
y :	2·50651	2·50893	2·51081	2·51188

(c) If $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$, $h = 0{\cdot}1$, find $y(0{\cdot}4)$ by using Euler's formula. Compare the result with analytical solution. 10

3. (a) Using Bisection method find a real root of $x^3 - x - 10 = 0$ to correct upto two decimal places. 5

(b) Calculate an approximate value of $\int_0^{\frac{\pi}{2}} \sin x \, dx$ by Simpson's $\frac{1}{3}$rd rule. After finding the true value of the integral, Compare with the approximate value of the integral. 5+2=7

(c) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = x^3 + \frac{y}{2}$ with $y(1) = 2$ at $x = 1{\cdot}1$ and $x = 1{\cdot}2$. 8

4. *(a)* Find the value of sin48° and sin58° from the following data : 8

x :	45°	50°	55°	60°
$\sin x$:	0·7071	0·7660	0·8192	0·8660

(b) Solve $\frac{dy}{dx} = \frac{1}{2}\left(1 + x^2\right)y^2$ with $y(0)=1$ at $x = 0{\cdot}1, 0{\cdot}2, 0{\cdot}3$ by an appropiate method and hence find $y(0{\cdot}4)$ by Milne's predictor-corrector method. 12

5. *(a)* Solve by Gauss-elimination method

$$4x - y + 2z = 15$$
$$-x + 2y + 3z = 5$$
$$5x - 7y + 9z = 8$$

5

(b) Using iteration method find a real root of the equation $x^3 + x - 1 = 0$ to correct upto four decimal places. 5

(c) Given that $\frac{dy}{dx} = x + y$ with the initial condition that $y = 0$ when $x = 0$. Find y for $x = 1$ by taking $h = 0{\cdot}2$ using modified Euler method. Give the correct upto four places of decimals. 10

6. *(a)* Solve the following equation by Gauss-Seidel method to correct upto four decimal places : 8

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

(b) Using Newton's divided difference formula find the form of the function from the following data : 6

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

(c) Using Newton-Raphson method find the real root of the equation $3x - \cos x - 1 = 0$ to correct upto four decimal places. 6

7. *(a)* What do you mean by curve fitting ? Fit a parabolic curve to the following data :

x :	-3	-1	1	3	4
y :	15	5	1	5	2

1+9=10

where x is the independent variable of the function y.

(b) Using Regula-Falsi method find a real root of the equation $\sin x + \cos x = 1$ to correct upto three decimal places. 6

(c) Find the first derivative of the function tabulated below at the point $x = 1{\cdot}5$.

x :	1·5	2·0	2·5	3·0	3·5	4·0
$f(x)$:	3·375	7.000	13·625	24·000	38·875	59·000

4