

Total number of printed pages-8

53 (IE 811) OPRE

2013

(May)

## OPERATION RESEARCH

Paper : IE 811

Full Marks : 100

Pass Marks : 30

Time : Three hours

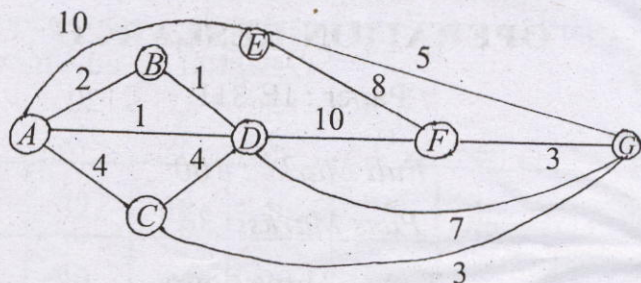
*The figures in the margin indicate full marks for the questions.*

*Answer any five questions.*

1. a) Describe a general L.P.P and explain how the constraints can be expressed as a linear system  $Ax=b$ ,  $x \geq 0$ , where symbols have their usual meaning. 2+3=5

Contd.

- b) Midwest TV cable company is in the process of providing cable service to seven new housing development areas. The cable miles are shown on each arc. Determine the most economical cable network. 5



- c) A company sells two different products A and B. The company makes a profit of Rs. 40 and Rs. 30 per unit on products A and B respectively. The two products are produced in a common production process and are sold in two different markets. The production process has a capacity of 30,000 man-hours. It takes 3 hours to produce one unit of A and one hour to produce one unit of B. The market has been surveyed and company officials feel that the maximum number of units of A that can be sold is 8,000 units and the maximum B is 12,000 units. Subject to these limitations, the products can be sold in any convex combination. Formulate a L.P.P. and solve that graphically. 5+5=10



2. a) Solve the following L.P.P. Simplex algorithm

$$\text{Max } Z = 4x_1 + 5x_2 + 9x_3 + 11x_4$$

Subject to

$$x_1 + x_2 + x_3 + x_4 \leq 15$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4 \leq 120$$

$$3x_1 + 5x_2 + 10x_3 + 15x_4 \leq 100$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

15

b) Find the dual of the primal problem

$$\text{Minimize } Zx = x_1 - 2x_2 + x_3$$

Subject to

$$3x_1 - x_2 + 5x_3 \leq 17$$

$$x_1 + 2x_2 - x_3 = 25$$

$$2x_1 - x_2 + 5x_3 \geq 57$$

$$x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \text{ is unrestricted.} \quad 5$$

3. a) A small project is composed of seven activities whose time estimates are listed in the table as follows: 10

Activity		Estimated duration (Weeks)		
i	j	Optimistic	Moot Likely	Pesimestic
1	2	1	1	7
1	3	1	4	7
1	4	2	2	8
2	5	1	1	1
3	5	2	5	14
4	6	2	5	8
5	6	3	6	15

- i) Draw the Project Network.  
 ii) Find the expected duration and variance of each activity. What is the expected project length?  
 iii) Calculate the variance and the standard deviation of the project length. What is the probability that the project will be completed ?  
 a) at least 4 weeks earlier than expected?  
 b) no more than 4 weeks later than expected?

(For standard normal  $Z=1.33$  area under the standard normal curve from  $O$  to  $Z$  is 0.4582).



- b) Write the mathematical model of an assignment problem. Discuss the Hungarian method for solving an assignment problem. 2+8=10

4. a) Solve the following L.P.P.

$$\text{Minimize } Zx = 4x_1 - 3x_2$$

Subject to

$$2x_1 - x_2 \geq 4$$

$$4x_1 + 3x_2 \leq 28; x_1, x_2 \geq 0$$

by duality theory and find the optimal solutions of the primal and dual as well as. 10

- b) Consider the set of equations

$$2x_1 - 3x_2 + 4x_3 + 6x_4 = 21$$

$$x_1 + 2x_2 + 3x_3 - 3x_4 + 5x_5 = 9$$

Note that  $x_1 = 2, x_2 = 1, x_3 = 2, x_4 = 2, x_5 = 1$  is a feasible solution. Reduce this to two different basic feasible solutions. 10

5. a) Define degeneracy. State the conditions for the occurrence of degeneracy in a L.P.P. Also explain under what conditions the B.F.S. of a given L.P.P. is optimal and the solution is unbounded. 2+4+2+2=10

3. a) A small project is composed of seven activities whose time estimates are listed in the table as follows: 10

Activity		Estimated duration (Weeks)		
i	j	Optimistic	Moot Likely	Pesimestic
1	2	1	1	7
1	3	1	4	7
1	4	2	2	8
2	5	1	1	1
3	5	2	5	14
4	6	2	5	8
5	6	3	6	15

- i) Draw the Project Network.
- ii) Find the expected duration and variance of each activity. What is the expected project length?
- iii) Calculate the variance and the standard deviation of the project length. What is the probability that the project will be completed ?
  - a) at least 4 weeks earlier than expected?
  - b) no more than 4 weeks later than expected?

(For standard normal  $Z=1.33$  area under the standard normal curve from  $O$  to  $Z$  is 0.4582).



- b) Write the mathematical model of an assignment problem. Discuss the Hungarian method for solving an assignment problem. 2+8=10

4. a) Solve the following L.P.P.

$$\text{Minimize } Zx = 4x_1 - 3x_2$$

Subject to

$$2x_1 - x_2 \geq 4$$

$$4x_1 + 3x_2 \leq 28; x_1, x_2 \geq 0$$

by duality theory and find the optimal solutions of the primal and dual as well as. 10

- b) Consider the set of equations

$$2x_1 - 3x_2 + 4x_3 + 6x_4 = 21$$

$$x_1 + 2x_2 + 3x_3 - 3x_4 + 5x_5 = 9$$

Note that  $x_1 = 2, x_2 = 1, x_3 = 2, x_4 = 2, x_5 = 1$  is a feasible solution. Reduce this to two different basic feasible solutions. 10

5. a) Define degeneracy. State the conditions for the occurrence of degeneracy in a L.P.P. Also explain under what conditions the B.F.S. of a given L.P.P. is optimal and the solution is unbounded. 2+4+2+2=10

b) What do you mean by critical path ?

A project has the following time schedule :

Activity	Duration (in weeks)	Activity	Duration (in weeks)
1-2	2	4-6	3
1-3	2	5-8	1
1-4	1	6-9	5
2-5	4	7-8	4
3-6	8	8-9	3
3-7	5		

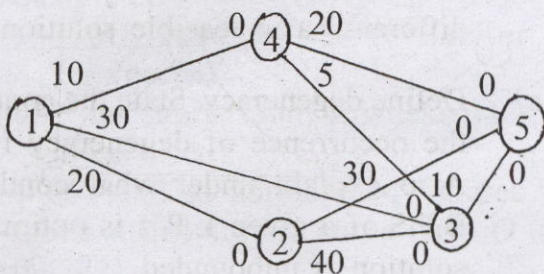
i) Draw the project diagram.

ii) Calculate the critical path and its duration.

iii) Determine independent floats.

$$2+5+3$$

6. a) Determine the minimum flow and calculate the amount of flow that occurs for each arc in the following network. 10



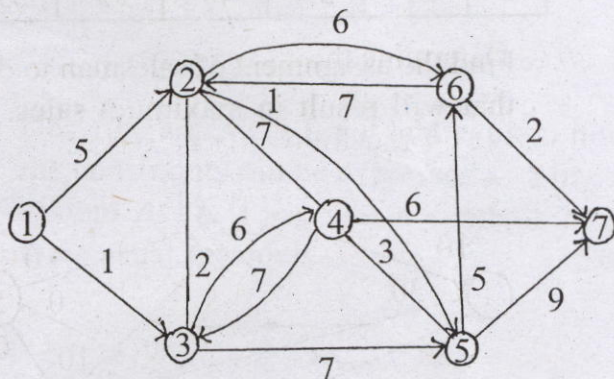


- b) Find the optimal solution to the following transportation problem in which the cells contain the transportation cost in rupees.

	$W_1$	$W_2$	$W_3$	$W_4$	$W_5$	Available
$F_1$	7	6	4	5	9	40
$F_2$	8	5	6	7	8	30
$F_3$	6	8	9	6	5	20
$F_4$	5	7	7	8	6	10
Required	30	30	15	20	5	100 (total)

10

7. a) Use Dijkstra's algorithm to find the shortest route between node 1 and every other node in the following network. 10



- b) A marketing manager has 5 salesman and 5 sales-districts. Considering the capabilities of the salesman and the nature of districts, the marketing manager estimates that sales per month (in hundred rupees) for each salesman in each district would be as follows :

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
1	32	38	40	28	40
2	40	24	28	21	36
Job 3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment of salesman to districts that will result in maximum sales. 10