Preliminary Studies on Injection Locking of Oscillators

A Project Work Submitted in Partial Fulfillment

of the requirements for the Degree of

BACHELOR OF TECHNOLOGY

in

ELECTRONICS & COMMUNICATION ENGINEERING

by

SANDEEP KUMAR PANDEY (GAU-C-11/13)
&
HEMANGA KALITA (GAU-C-11/124)
&
ANJUMANI SAIKIA (GAU-C-11/25)
&
UJJAL CHOUDHURY(GAU-C-11/L-180)
&
RAJARAM CHANDRA SAH(GAU-C-11/L-176)

Under the supervision of

Mr.Arindum Mukherjee

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

केन्द्रीय प्रौद्योगिकी संस्थान कोकराझार

CENTRAL INSTITUTE OF TECHNOLOGY KOKRAJHAR

(A Centrally Funded Institute under Ministry of HRD, Govt. of India)

BODOLAND TERRITORIAL AREAS DISTRICTS :: KOKRAJHAR :: ASSAM :: 783370

Website: www.cit.kokrajhar. in, www.cit.ac.in

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

केन्द्रीय प्रौद्योगिकी संस्थान कोकराझार

CENTRAL INSTITUTE OF TECHNOLOGY, KOKRAJHAR

(An Autonomous Institute under MHRD)

Kokrajhar – 783370, BTAD, Assam, India

CERTIFICATE OF APPROVAL

This is to certify that the work embodied in this project entitled **Preliminary Studies On Injection Locking of Oscillators** submitted by Sandeep Kumar Pandey, Hemanga Kalita, Anjumani Saikia, Ujjal Choudhury, Rajaram Chandra Sah to the Department of Electronics & Communication Engineering, is carried out under our direct supervisions and guidance.

The project work has been prepared as per the regulations of Central Institute of Technology and I strongly recommend that this project work be accepted in partial fulfillment of the requirement for the degree of B.Tech.

Supervisor

(Mr.Arindum Mukherjee)

Assistant Professor, Dept. of ECE

Countersigned by

(Ms.Namita Das)

HoD (i/c)

Department of ECE

HoD Incharge
E.C.E. Department,
CIT, Kokrajhar

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

केन्द्रीय प्रौद्योगिकी संस्थान कोकराझार

CENTRAL INSTITUTE OF TECHNOLOGY, KOKRAJHAR

(An Autonomous Institute under MHRD)

Kokrajhar – 783370, BTAD, Assam, India

Certificate by the Board of Examiners

This is to certify that the project work entitled **Preliminary Studies on Injection Locking of Oscillators** submitted by Sandeep Kumar Pandey, Hemanga Kalita, Anjumani Saikia, Ujjal Choudhury, Rajaram Chandra Sah to the Department of Electronics & Communication Engineering of Central Institute of Technology, Kokrajhar has been examined and evaluated.

The project work has been prepared as per the regulations of Central Institute of Technology and qualifies to be accepted in partial fulfillment of the requirement for the degree of B. Tech.

Project Co-ordinator

Board of Examiners

20/5/15

ACKNOWLEDGEMENT

The satisfaction that accompanies the successful completion of any task would be incomplete without the mention of people whose ceaseless cooperation made it possible, whose constant guidance and encouragement crown all efforts with success.

We bestow our hearted appreciation and gratefulness to the efforts made by our project guide: Asst. Professor **Arindum Mukherjee**, in terms of the Project Idea, instruction, suggestions and the opportunity, without which our efforts would have remained astray.

We do gratefully acknowledge and express our gratitude to all staff members of *Electronics and Communication Engineering* and friends who supported us in preparing this project.

Date: 20/05/2015

Sandeep Kumar Pandey
SANDEEP KUMAR PANDEY
University Roll: GAU-C-11/13
University Registration No: 014330 of 2011-12

Hemanga Kalita
HEMANGA KALITA
University Roll: GAU-C-11/124
University Registration No: 015077 of 2011-12

Anjumani Saikia
ANJUMANI SAIKIA
University Roll: GAU-C-11/25
University Registration No: 014389 of 2011-12

Ujjal Choudhury
UJJAL CHOUDHURY
University Roll: GAU-C-11/L-180
University Registration No: 084551 of 2012-13

Rajaram Chandra Sah
RAJARAM CHANDRA SAH
University Roll: GAU-C-11/L-176
University Registration No: 084547 of 2012-13

TABLE OF CONTENTS

List of figures

Page no

List of Tables

LITERATURE SURVEY

This project is based on the frequency locking phenomenon exhibited by the non-linear oscillators. There have been a lot of research and work previously done on this topic. Various approaches are used by various scholars and scientists to arrive at conclusions that injection locking is observed in case of non-linear oscillators. Some of them are mentioned below:-

1) . B. Razavi, in his paper "A study of injection locking and pulling in oscillators," have derived injection locking characteristics of oscillators and presented a graphical analysis that describes injection pulling in time and frequency domain.

2) R. Adler, in his paper "A study of locking phenomena in oscillators," Using the assumption that time constants in the oscillator circuit are small compared to the length of one beat cycle ,have derived a differential equation which gives the oscillator phase as a function of time. With the aid of this equation, the transient process of "pull-in "as well as the production of a distorted beat note are described in detail.

3) I.Ali , A. Banerjee, A. Mukherjee and B.N. Biswas, in their paper "Study of Injection Locking with Amplitude Perturbation and its effect on Pulling of Oscillators", have studied the Injection locking characteristics of oscillators both qualitatively and analytically and the closed-form expressions of frequency-pulling and spectrum of the unlocked driven oscillator is estimated with negligible amplitude perturbation. Modifications in spectrum, lock range, and pulling of the oscillator are shown under significant amplitude perturbation.

Achievement

International Journal of Electronics and Communication Technology (IJECT), Preliminary studies on injection locking of oscillators, volume-6 issue 1 spl-1 pg 156-159.

INTRODUCTION

Oscillator:-

An oscillator provides a source of repetitive A.C. signal across its output terminals without needing any input (except a D.C. supply). The signal generated by the oscillator is usually of constant amplitude.
The wave shape and amplitude are determined by the design of the oscillator circuit and choice of component values.
The frequency of the output wave may be fixed or variable, depending on the oscillator design.

Types of Oscillator

Oscillators may be classified by the type of signal they produce.

- **SINE WAVE OSCILLATORS**

These circuits ideally produce a pure sine wave output having a constant amplitude and stable frequency. The type of circuit used depends on a number of factors, including the frequency required. Designs based on LC resonant circuits or on crystal resonators are used for ultrasonic and radio frequency applications, but at audio and very low frequencies the physical size of the resonating components, L and C would be too big to be practical.
For this reason a combination of R and C is used to a control frequency. The circuit symbols used for these frequency control networks are shown in Fig. 1.0.2

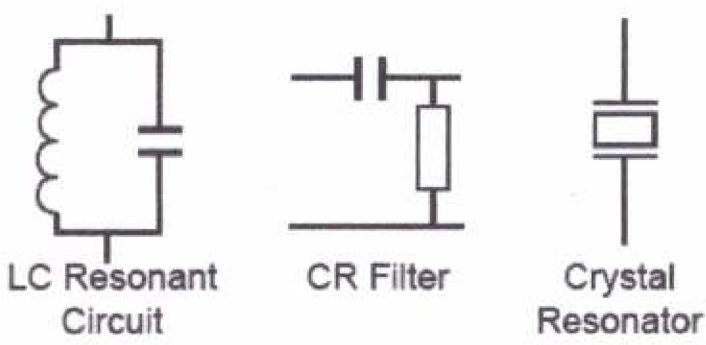

Fig 1:- Frequency Control Networks

- **RELAXATION OSCILLATORS and ASTABLE MULTIVIBRATORS .**

These oscillators work on a different principle to sine wave oscillators. They produce a square wave or pulsed output and generally use two amplifiers, and a frequency control network that simply produces a timing delay between two actions. The two amplifiers operate in switch mode, switching fully on or fully off alternately, and as the time, during which the transistors are actually switching, only lasts for a very small fraction of each cycle of the wave, the rest of the cycle they "relax" while the timing network produces the remainder of the wave. An alternative name for this type of oscillator is an "astable multivibrator", this name comes from the fact that they contain more than one oscillating element. There are basically two oscillators, i.e. "vibrators", each feeding part of its signal back to the other, and the output changes from a high to a low state and back again continually, i.e. it has no stable state, hence it is astable. Relaxation oscillators can be built using several different designs and can work at many different frequencies. Astables may typically be chosen for such tasks as producing high frequency digital signals. They are also used to produce the relatively low frequency on-off signals for flashing lights.

- **SWEEP OSCILLATORS**

A sweep waveform is another name for a saw-tooth wave. This has a linearly changing (e.g increasing) voltage for almost the whole of one cycle followed by a fast return to the wave's original value. This wave shape is useful for changing (sweeping) the frequency of a voltage-controlled oscillator, which is an oscillator that can have its frequency varied over a set range by having a variable 'sweep' voltage applied to its control input. Sweep oscillators often consist of a ramp generator that is basically a capacitor charged by a constant value of current. Keeping the charging current constant whilst the charging voltage increases, causes the capacitor to charge in a linear fashion rather than its normal exponential curve. At a given point the capacitor is rapidly discharged to return the signal voltage to its original value. These two sections of a saw-tooth wave cycle are called the sweep and the fly-back.

Oscillator Operation

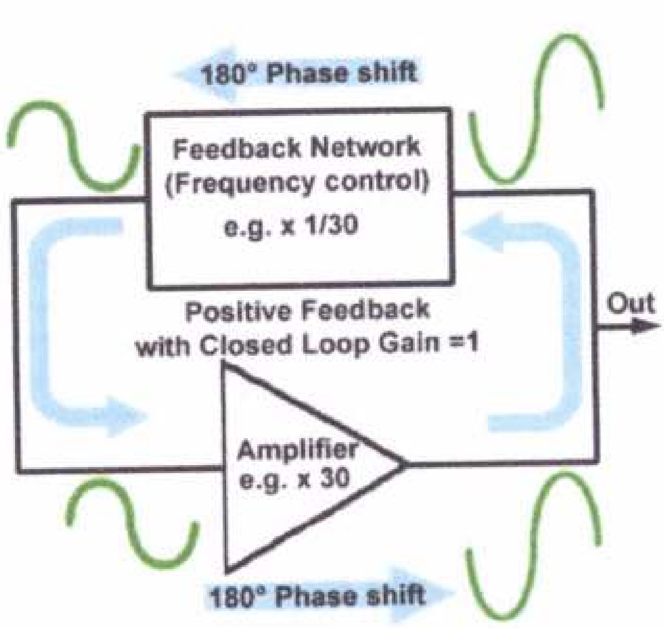

Fig 2:- The Essential Elements of an Oscillator

Parts of an Oscillator

Most oscillators consist of three basic parts:

1. An amplifier. This will usually be a voltage amplifier and may be biased in class A, B or C.

2. A wave shaping network. This consists of passive components such as filter circuits that are responsible for the shape and frequency of the wave produced.

3. A POSITIVE feedback path. Part of the output signal is fed back to the amplifier input in such a way that the feed back signal is regenerated, re-amplified and fed back again to maintain a constant output signal.

The conditions for oscillation

Positive feedback must occur at a frequency where the voltage gain of the amplifier is equal to the losses (attenuation) occurring in the feedback path. For example if 1/30th of the output signal is fed back to be in phase with the input at a particular frequency, and the gain of the amplifier (without feedback) is 30 times or more, oscillation will take place.

The oscillations should take place at one particular frequency.

The amplitude of the oscillations should be constant.

There are many different oscillator designs in use, each design achieving the above criteria in different ways. Some designs are particularly suited to producing certain wave shapes, or work best within a certain band of frequencies. Whatever design is used however, the way of achieving a signal of constant frequency and constant amplitude is by using one or more of three basic methods

Method 1

Make sure that positive feedback occurs only at one frequency, the required frequency of oscillation. This may be achieved by ensuring that only signals of the required frequency are fed back, or by ensuring the feedback signal is in the correct phase at only one frequency.

Method 2

Make sure that sufficient amplification for oscillation can take place only at the required frequency, by using an amplifier that has an extremely narrow bandwidth, extending to the frequency of oscillation only.

Method 3

Use amplifiers in "switch mode" to switch the output between two set voltage levels, together with some form of time delay to control the time at which the amplifiers switch on or off, thus controlling the periodic time of the signal produced.

Methods 1 and 2 are used extensively in sine wave oscillators, while method 3 is useful in square wave generators, sometimes called aperiodic (untuned) oscillators. Oscillators using method 3 often use more than one amplifier and timing circuit, and so are called multivibrators (more than one oscillator).

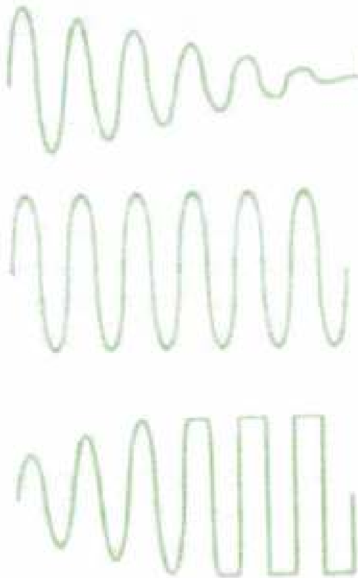

Fig 3:- The need for amplitude stability

Barkhausen Criterion

The frequency of oscillation at which sinusoidal oscillator operates is the frequency for which the total shift introduced, as the signal proceeds from the input terminals, through the amplifier and feedback network, and back again to the input, is precisely zero(or an integral multiple of 2*Π).

(Or)

Stated simply the condition $A^*\beta = -1$ at $\omega = \omega o$, i.e. the magnitude of loop gain should be one and phase of loop gain should be unity (the feedback network introduces 180 degree phase shift, the other 180 degree phase shift is provided by mixer) is called Barkhausen criterion.

Overview of Injection locking of oscillators

A self oscillating system or a self-oscillator is a primary source of oscillations operating under self-exciting conditions. Any oscillator is a nonlinear system converting the dc power of a source into ac energy. The nonlinearity of the system (curvature of the current-voltage characteristic of the amplifying element) begins to manifest itself with the rise of the amplitudes, and consequently the gain of the system drops. The amplitudes stop increasing when the amplification is reduced to the level at which the damping of the oscillations in the load circuit is balanced; in this case the energy supplied by the amplifier per period proves to be equal to the energy consumed in the circuit load during the same time.

An oscillator operating under steady-state conditions is a conventional nonlinear amplifier excited by the oscillations produced in the oscillator itself; these oscillations are fed from the oscillatory system of the amplifier and applied to its input through a feedback loop. When the amplitude and phase of oscillations satisfy certain conditions, the behavior of a self-oscillator is identical to that of a separately excited oscillator. In the last stage of the transient conditions the behavior of the oscillator is determined mainly by the nonlinear character of the system, so that the steady state of the oscillator cannot be described without taking the nonlinearity into account.

Injection locking becomes useful in a number of applications including frequency division, quadrature generation, and oscillators with finer phase separations. However, injection pulling on the other hand typically proves undesirable. For example, in a broadband transceiver, the two voltage controlled oscillators may pull each other as a result of substrate coupling.

In this report, we have derived the locking equation for an injection synchronized oscillator using two approaches. The first method is based on simple phasor algebra and the second one is derived using the conventional circuit theory approach.

Band Pass Filters

Band Pass Filters

The cut-off frequency or fc point in a simple RC passive filter can be accurately controlled using just a single resistor in series with a non-polarized capacitor, and depending upon which way around they are connected, we have seen that either a Low Pass or a High Pass filter is obtained.

One simple use for these types of Passive Filters is in audio amplifier applications or circuits such as in loudspeaker crossover filters or pre-amplifier tone controls. Sometimes it is necessary to only pass a certain range of frequencies that do not begin at 0Hz, (DC) or end at some upper high frequency point but are within a certain range or band of frequencies, either narrow or wide.

By connecting or "cascading" together a single Low Pass Filter circuit with a High Pass Filter circuit, we can produce another type of passive RC filter that passes a selected range or "band" of frequencies that can be either narrow or wide while attenuating all those outside of this range. This new type of passive filter arrangement produces a frequency selective filter known commonly as a Band Pass Filter or BPF for short.

Band Pass Filter Circuit

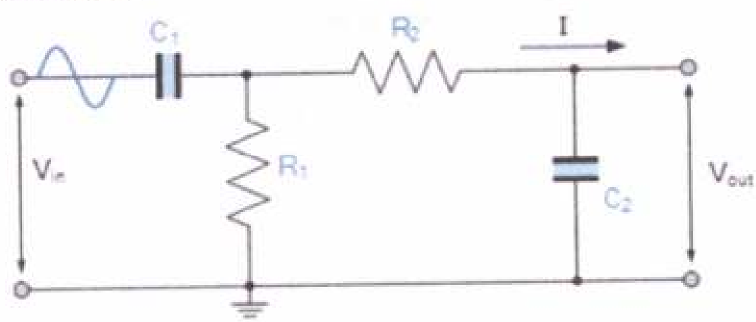

Fig 4:- band pass filter circuit

Unlike a **low pass filter** that only pass signals of a low frequency range or a **high pass filter** which pass signals of a higher frequency range, a **Band Pass Filters** passes signals within a certain "band" or "spread" of frequencies without distorting the input signal or introducing extra noise. This band of frequencies can be any width and is commonly known as the filters **Bandwidth**.

Bandwidth is commonly defined as the frequency range that exists between two specified frequency cut-off points (fc), that are 3dB below the maximum centre or resonant peak while attenuating or weakening the others outside of these two points.

Then for widely spread frequencies, we can simply define the term "bandwidth", BW as being the difference between the lower cut-off frequency (fc_{LOWER}) and the higher cut-off frequency (fc_{HIGHER}) points. In other words, BW = $f_H - f_L$. Clearly for a pass band filter to function correctly, the cut-off frequency of the low pass filter must be higher than the cut-off frequency for the high pass filter.

The "ideal" **Band Pass Filter** can also be used to isolate or filter out certain frequencies that lie within a particular band of frequencies, for example, noise cancellation. Band pass filters are known generally as second-order filters, (two-pole) because they have "two" reactive component, the capacitors, within their circuit design. One capacitor in the low pass circuit and another capacitor in the high pass circuit.

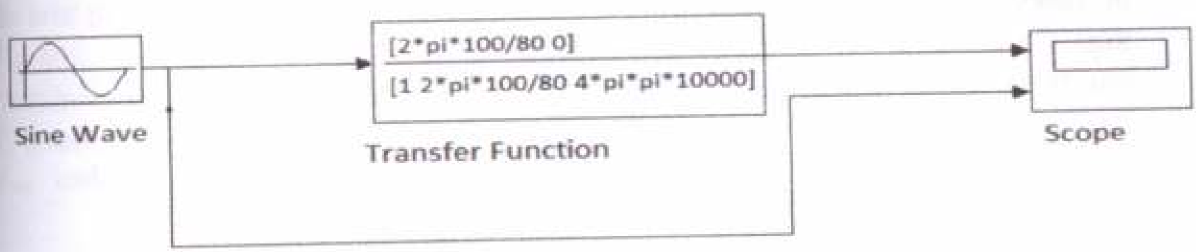

Fig 5 :- Matlab Simulink Setup for a Band Pass Filter

In the above band filter we take the vale of Q arbitrarily and after the experiment we have to find Q using the formula which is given below:-

$$Q = \omega / \Delta\omega$$

Where $\Delta\omega$=Bandwith

The transfer function for the Bandpass Filter is assumed to be a Second Order Transfer Function given by:-

$$H(S) = \frac{(W/Q)S}{S^2 + (W/Q)S + W^2}$$

Where $W = 2*\pi*100$

$Q = 80$

Frequency shifting arrangement by injection synchronization

Consider the simple (conceptual) oscillator shown in Fig.6, where all parasitics are neglected, the tank operates at the res-onance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ (thus contributing no phase shift), and the ideal inverting buffer follows the tank to create a total phase shift of $360°$ around the feedback loop. What happens if an additional phase shift is inserted in the loop, e.g., as depicted in Fig. 6(b)? The circuit can no longer oscillate at ω_0 at because the total phase shift at this frequency deviates from $360°$ by ϕ_0 . Thus, as illustrated in Fig. 6(c), the oscillation frequency must change to a new value such that the tank contributes enough phase shifts to cancel the effect of ϕ_0 . Note that, if the buffer and T_1 contribute no phase shift, then the drain current of T_1 (I_{osc}) must remain in phase with V_{out} under all conditions.

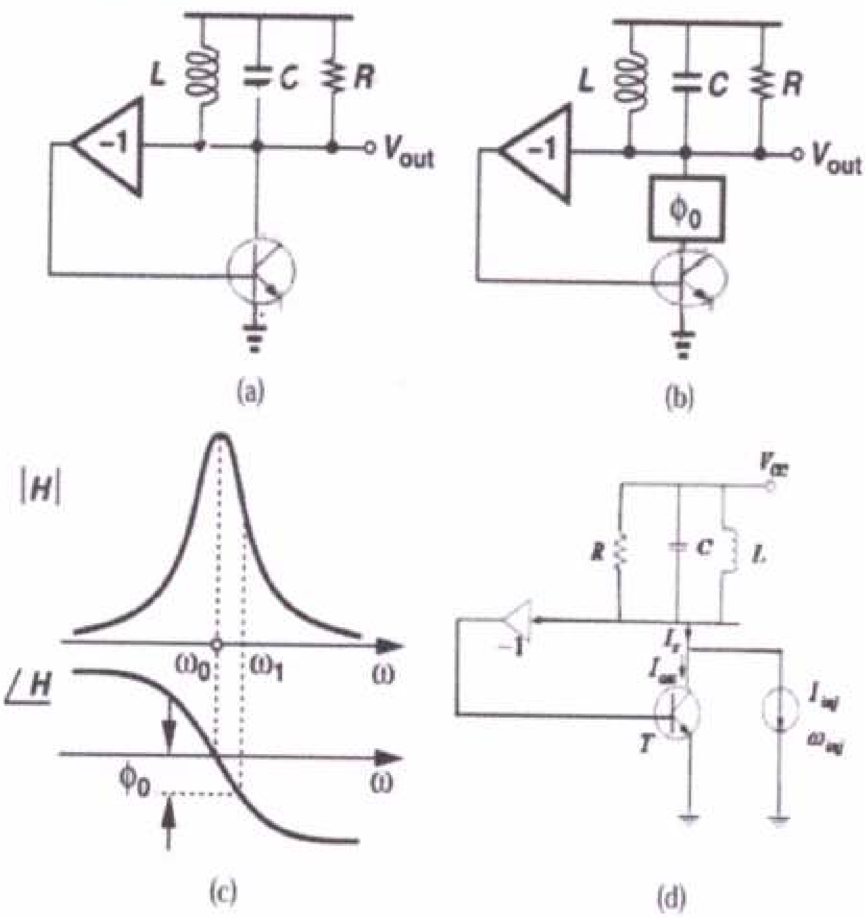

Fig 6:- (a) Conceptual oscillator. (b) Frequency shift due to additional phase shift. (c) Open-loop characteristics. (d) Frequency shift by injection.

Now suppose we attempt to produce ϕ_0 by adding a sinusoidal current to the drain current of T_1 [Fig. 6(d)]. If the amplitude and frequency of I_{inj} are chosen properly, the circuit indeed oscillates at ω_{inj} rather than at ω_0 and injection locking occurs. Under this condition, V_{out} and I_{inj} must bear a phase difference [Fig.6] because:

1) the tank contributes phase at $\omega_{inj} \neq \omega_0$, rotating V_{out} with respect to the resultant current I_T,, and

2) I_{osc} still remains in phase with V_{out} and hence out of phase with respect to I_T , requiring that I_{inj} form an angle with $I_{osc.}$ (If I_{inj} and $I_{osc.}$ were in phase, then I_T would also be in phase with I_{osc} and thus with V_{out}). The angle formed between I_{osc} and I_{inj} is such that I_T becomes aligned with V_{osc} (and I_{osc})after experiencing the tank phase shift ϕ_0, at ω_{inj} .

DERIVATION OF THE LOCKING EQUATION

Locking range refers to the range of frequencies of 'ω_{inj}' across which injection locking holds. To match the increasingly greater phase shift introduced by the tank, the angle between the 'I_{osc}' and 'I_T' must also increase, requiring that 'I_{osc}' will rotate anticlockwise. Using trigonometric identity, it is not difficult to show that

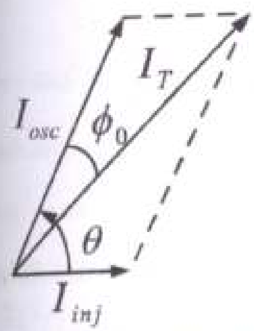

Fig. 7 Phasor diagram of oscillator

$$\frac{I_{inj}}{\sin\phi_0} = \frac{I_{osc}}{\sin(\theta-\phi_0)} = \frac{I_T}{\sin(\pi-\theta)}$$

$$\frac{I_{osc}}{I_{inj}} = \frac{\sin(\theta-\phi_0)}{\sin\phi_0} = \sin\theta.\cot\phi_0 - \cos\theta \qquad (1)$$

Again,

$$\sin\phi_0 = \frac{I_{inj}}{I_T}\sin\theta$$

$$\frac{I_{osc} + I_{inj}\cos\theta}{I_{inj}} = \sin\theta\cot\phi_0$$

$$\Rightarrow \tan\phi_0 = \frac{I_{inj}\sin\theta}{I_{osc} + I_{inj}\cos\theta} \qquad (2)$$

A second-order parallel tank circuit consisting of 'L', 'C' and 'R' exhibits an impedance [7] of $Z(j\omega)=\frac{1}{\frac{1}{R}+j\left(\omega C-\frac{1}{\omega L}\right)}=\frac{\frac{1}{R}-j\left(\omega C-\frac{1}{\omega L}\right)}{\frac{1}{R^2}+\left(\omega C-\frac{1}{\omega L}\right)^2}$.

Thus the phase shift introduced by the tank circuit near resonance is given by

$$\phi=-\tan^{-1}\left[\frac{\omega C-\frac{1}{\omega L}}{1/R}\right]=-\tan^{-1}\left[\frac{R}{\omega L}\left(\omega^2 LC-1\right)\right]$$

$$=-\tan^{-1}\left[\frac{Q}{\omega_0^{\,2}}\left(\omega^2-\omega_0^{\,2}\right)\right]\approx-\tan^{-1}\left[\frac{2Q}{\omega_0}\left(\omega-\omega_0\right)\right]$$

where the following simplifications have been used : $\omega_0^2-\omega^2\approx 2\omega_0\left(\omega_0-\omega\right)$; $Q=\frac{R}{\omega L}$ and $\omega_0=\frac{1}{\sqrt{LC}}$. Hence equating (2) with the phase shift introduced by the circuit, one gets

$$\frac{2Q}{\omega_0}\left[\left(\omega-\omega_L\right)-\left(\omega_0-\omega_L\right)\right]=-\frac{I_{inj}\sin\theta}{I_{osc}+I_{inj}\cos\theta}$$

$$\frac{2Q}{\omega_0}\left[\frac{d\theta}{dt}-\Delta\omega_0\right]=-\frac{I_{inj}\sin\theta}{I_{osc}+I_{inj}\cos\theta}$$

$$\frac{d\theta}{dt}=\Delta\omega_0-\frac{\omega_0}{2Q}\left(\frac{I_{inj}}{I_{osc}}\right)\frac{\sin\theta}{1+\left(\frac{I_{inj}}{I_{osc}}\right)\cos\theta} \qquad (3)$$

Since, $\frac{I_{inj}}{I_{osc}}\ll 1$, (3) reduces to $\frac{d\theta}{dt}=\Delta\omega_0-\frac{\omega_0}{2Q}\left(\frac{I_{inj}}{I_{osc}}\right)\sin\theta$ (4)

where '$\Delta\omega_0$' is open-loop frequency error and (4) is the famous Adler's equation [6].

OSCILLATOR UNDER WEAK INJECTION

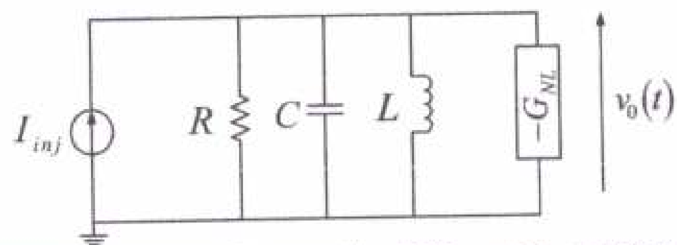

Fig. 8 Equivalent circuit diagram of a negative differential conductance oscillator with injection

In the following analysis, we take a negative differential conductance oscillator in presence of an injection signal '$I_{inj}(t)$'. Here, 'R' accounts for the losses in the tank circuit and we model the oscillator as a one-port circuit consisting of a parallel tank circuit and a non-linear element 'G_{NL}'. Since *a linear oscillator does not injection lock*, the non-linearity in the circuit will aid the process of injection locking. Typically the non-linearity in Fig. 3 arises because of the non-linearity present in the active core of the transistor [8] in Fig.1.

Application of Kirchhoff's current law, one gets

$$\frac{dv_0}{dt}=\frac{I_{inj}}{C}-\frac{1}{LC}\int v_0 dt-\frac{v_0}{RC}+\frac{G_{NL}}{C}v_0$$

$$C\frac{d^2v_0}{dt^2}+\frac{v_0}{L}-\left[G_{NL}-\frac{1}{R}\right]\frac{dv_0}{dt}=\frac{dI_{inj}}{dt} \qquad (5)$$

The output of the oscillator and the synchronizing signal are taken as $I_{inj}=I_0e^{j\omega_{inj}t}$ and $v_0=V(t)e^{j[\omega_{inj}t+\theta(t)]}$; where '$V(t)$' is the envelope of the oscillator output and '$\theta(t)$' is the output phase modulation because of the synchronizing signal .

$$\therefore \frac{dv_0}{dt}=e^{j[\omega_{inj}t+\theta(t)]}\left[\frac{dV}{dt}+j\left(\omega_{inj}+\frac{d\theta}{dt}\right)V(t)\right]$$

$$\frac{d^2v_0}{dt^2}=\left\{\begin{matrix} j\left(\omega_{inj}+\frac{d\theta}{dt}\right)\left[\frac{dV}{dt}+j\left(\omega_{inj}+\frac{d\theta}{dt}\right)V(t)\right]+ \\ \left[\frac{d^2V}{dt^2}+j\left(\omega_{inj}+\frac{d\theta}{dt}\right)\frac{dV}{dt}+j\frac{d^2\theta}{dt^2}V(t)\right] \end{matrix}\right\}e^{j[\omega_{inj}t+\theta(t)]}$$

and

$$\frac{dI_{inj}}{dt}=j\omega_{inj}I_0e^{j\omega_{inj}t}=j\omega_{inj}I_0e^{j[\omega_{inj}t+\theta(t)]}e^{-j\theta(t)}$$

Using these results in (5), one gets

$$C\left[\begin{array}{l}\frac{d^2V}{dt^2}+j\frac{d^2\theta}{dt^2}V(t)+2j\left(\omega_{inj}+\frac{d\theta}{dt}\right)\frac{dV}{dt}\\-\left(\omega_{inj}+\frac{d\theta}{dt}\right)^2V(t)\end{array}\right]$$

$$+\frac{1}{L}V(t)-\left[G_{NL}-\frac{1}{R}\right]\left[\frac{dV}{dt}+j\left(\omega_{inj}+\frac{d\theta}{dt}\right)V(t)\right]$$

$$=\omega_{inj}I_0\left(\sin\theta+j\cos\theta\right)$$

Equating the real and imaginary parts, it is not difficult to show that

$$C\left[\frac{d^2V}{dt^2}-\left(\omega_{inj}+\frac{d\theta}{dt}\right)^2V(t)\right]+\left(\frac{1}{R}-G_{NL}\right)\frac{dV}{dt}$$
$$+\frac{V(t)}{L}=\omega_{inj}I_0\sin\theta \qquad (6)$$

and

$$2C\left(\omega_{inj}+\frac{d\theta}{dt}\right)\frac{dV}{dt}+CV(t)\frac{d^2\theta}{dt^2}+$$
$$\left(\frac{1}{R}-G_{NL}\right)\left(\omega_{inj}+\frac{d\theta}{dt}\right)V(t)=\omega_{inj}I_0\cos\theta \qquad (7)$$

Now the following assumptions are made in order to simplify (6) and (7) : (i) since '$V(t)$' and '$\theta(t)$' are slowly varying functions of time, $\frac{1}{V(t)}\left(\frac{dV}{dt}\right)\ll 1$ and $\frac{1}{\omega_0}\left(\frac{d\theta}{dt}\right)\ll 1$, (ii) the magnitude of the oscillator amplitude envelope is approximately the tank peak current produced by the '$-G_{NL}$' multiplied by the tank resistance 'R', i.e., $V|_{max}=I_{osc}\times R=I_{osc}\times Q\omega_0 L$,

(iii) $\omega_0^2-\omega_{inj}^2\approx 2\omega_0\left(\omega_0-\omega_{inj}\right)$, and (iv) $\omega_{inj}\approx\omega_0$.
Hence, (7) can be written as

$$\left(\frac{1}{R}-G_{NL}\right)\left(\omega_{inj}+\frac{d\theta}{dt}\right)V(t)+2C\omega_{inj}\frac{dV}{dt}=\omega_{inj}I_0\cos\theta$$

i.e., $\frac{dV}{dt}+\frac{1}{2C}\left(\frac{1}{R}-G_{NL}\right)V(t)=\frac{I_0}{2C}\cos\theta$ (8)

It is worthwhile to mention that (8) gives the behavior of the envelope of the oscillator output. Further, (6) gives

$$-C\left[\omega_{inj}^{2}+2\omega_{inj}\frac{d\theta}{dt}\right]+\left(\frac{1}{R}-G_{NL}\right)\frac{1}{V(t)}\frac{dV}{dt}$$

$$+\frac{1}{L}=\frac{\omega_{inj}I_0\sin\theta}{V(t)}$$

and substituting $\frac{1}{L}=\omega_0^2C$ one gets

$$\left(\omega_0^2-\omega_{inj}^2\right)-2\omega_{inj}\frac{d\theta}{dt}=\frac{\omega_{inj}I_0\sin\theta}{I_{osc}Q\omega_0LC}$$

i.e., $\frac{d\theta}{dt}=\left(\omega_0-\omega_{inj}\right)-\frac{\omega_0 I_0}{2QI_{osc}}\sin\theta$ (9)

Eqn. (9) is the Adler's equation derived in (4) by a somewhat different approach. In the steady state, $\frac{d\theta}{dt}=0\Rightarrow\frac{\left(\omega_0-\omega_{inj}\right)}{\frac{\omega_0}{2QI_{osc}}}=I_0\sin\theta$ gives the ideal locking or synchronizing range of the oscillator.

Matlab Simulink set up

Parallel Tuned Circuit with Non-linearity (Conductance)

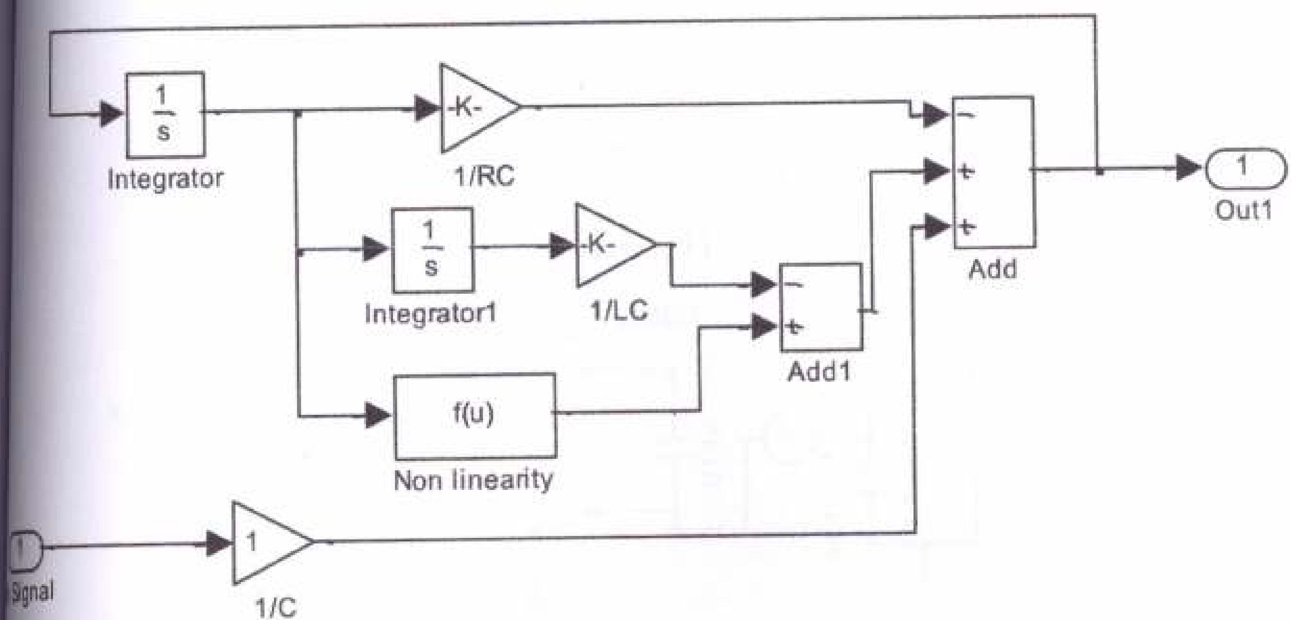

Fig 9.

Free Running Frequency= 45 Hz

Tina-Ti simulation set up

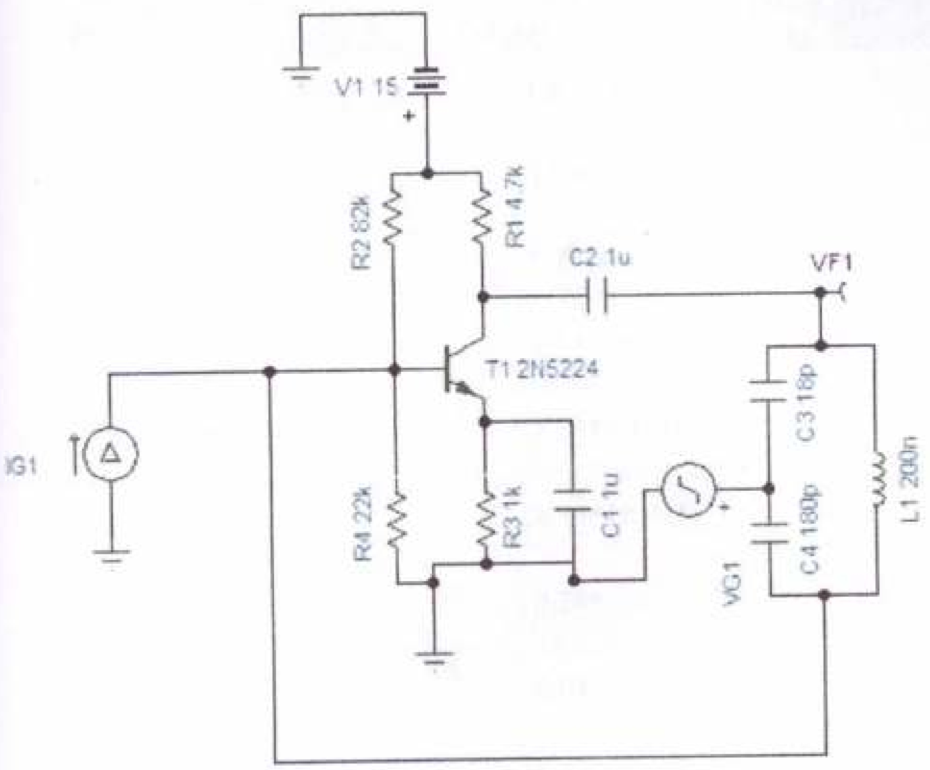

Fig 10

Free Running Frequency= 73 Hz

RESULTS AND DISCUSSIONS

The simulated values for band pass filter is represented in tabular form below, alongwith the the frequency response of the Band Pass filter.

Freqency	Output
5	1.87*10^(-4)
10	3.788*10^(-4)
15	5.7*10^(-4)
30	1.23*10^(-3)
50	2.5*10^(-3)
80	8.33*10^(-3)
100	0.285
110	0.01
125	8.32*10^(-3)
140	5.4*10^(-3)
150	4.3*10^(-3)
160	3.8*10^(-3)
180	2.8*10^(-3)
200	2*10^(-3)

Table 1:- Simulated values of band pass filter output at different frequencies.

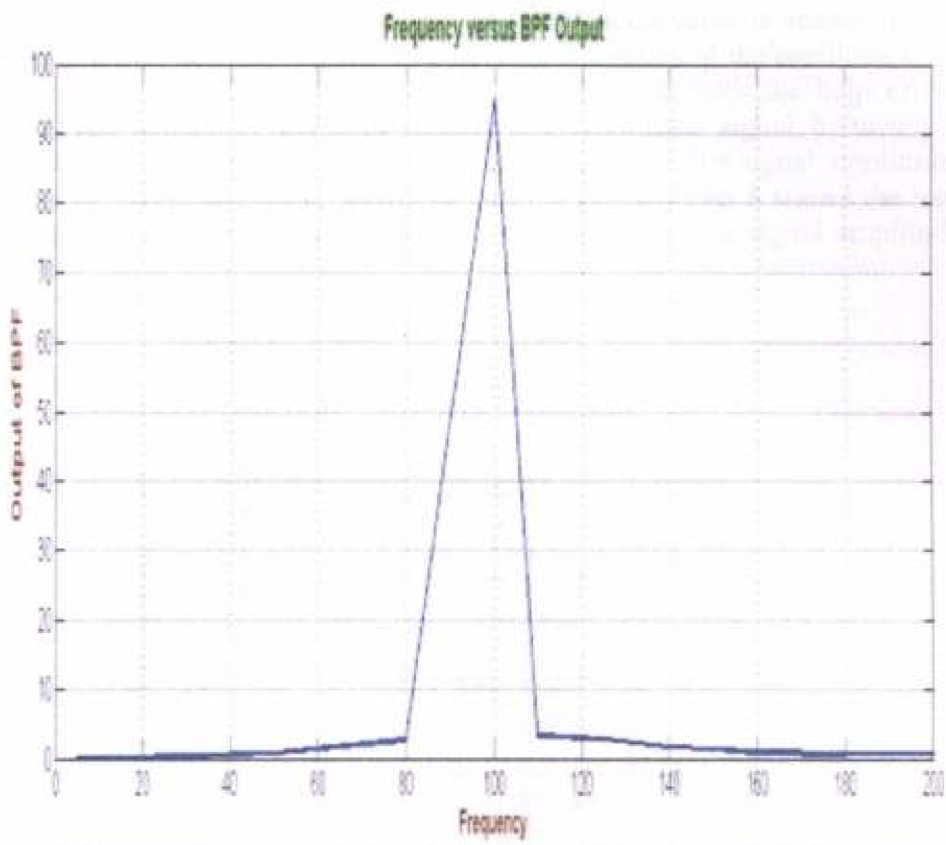

Fig 11 :- Frequency response of band pass filter

The oscillator output phase variation and its steady state value is shown in Fig. 5. The result is obtained by solving (9). The free running frequency of the oscillator is 45 Hz and is shown in Fig. 6. The experimental validation is done with the help of MATLAB SIMULINK and is shown in Fig. 4. In Fig. 7 the injection signal frequency is varied (hence the frequency detuning) and the corresponding injection signal amplitude is noted at the verge of synchronization, shown in table-1. Finally, Fig. 8 shows the variation of locking range with the frequency detuning when the injection signal amplitude is kept fixed at 150.

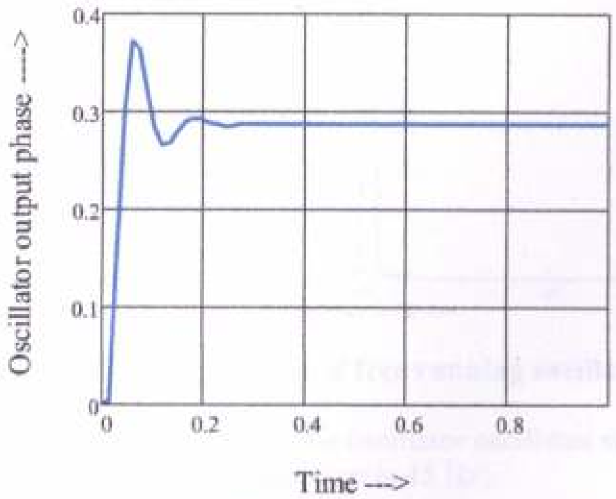

Fig. 12 :- Oscillator output phase variation

Injection locking phenomenon represented via spectrums

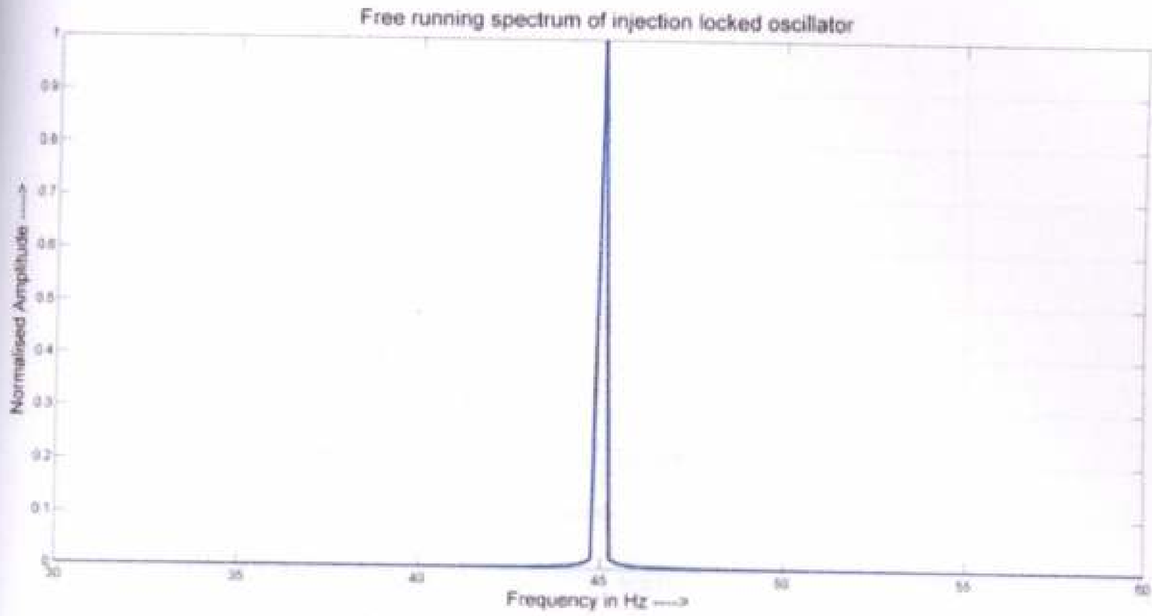

Fig. 13.1. Spectrum of free running oscillator

When there are no injection signal then the oscillator oscillates with free running frequency(in our case free running frequency is 45 Hz).

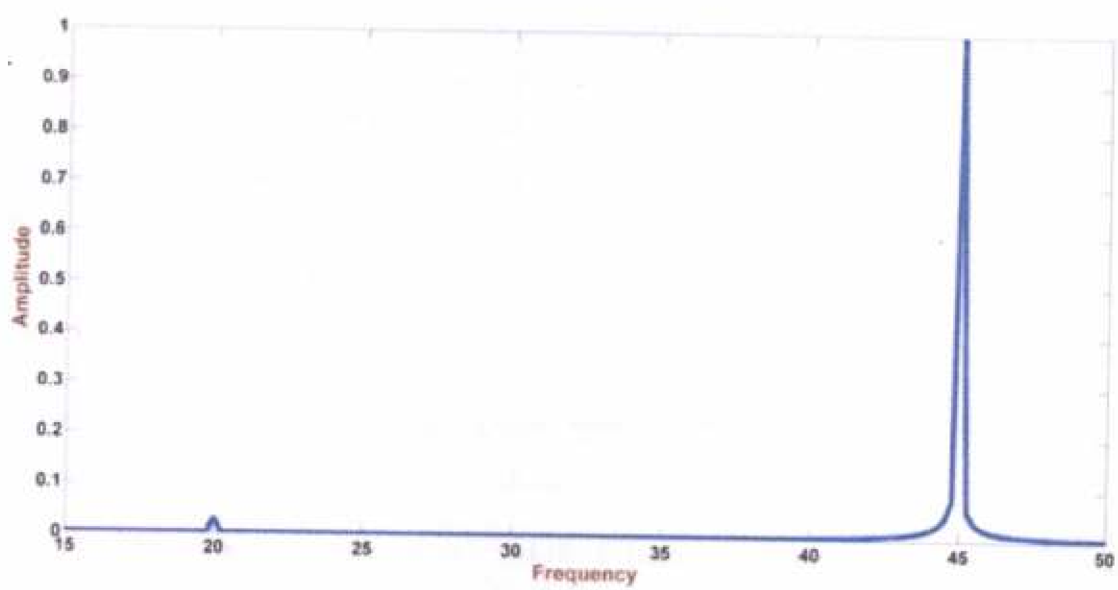

Fig 13.2 :- Illustration of the mechanics of the injection locking phenomena with low injection frequency

Amplitude= 40 & Frequency= 20 Hz

Oscillator doesn't lock at this frequency and it oscillates at the free-running frequency

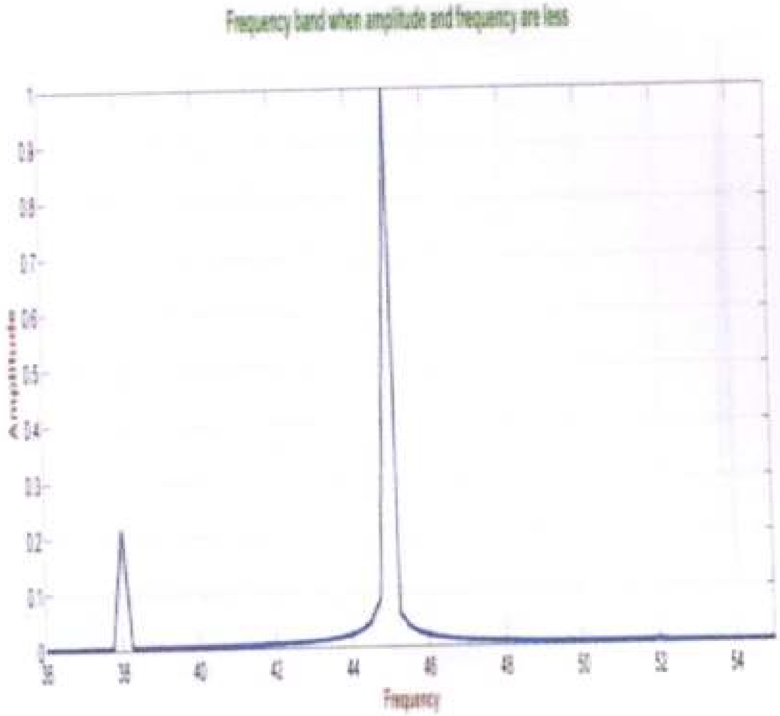

Fig 13.3:- Spectrum of external signal with low frequency

When injection frequency and amplitude are small (freq=38 Hz & amp = 40).

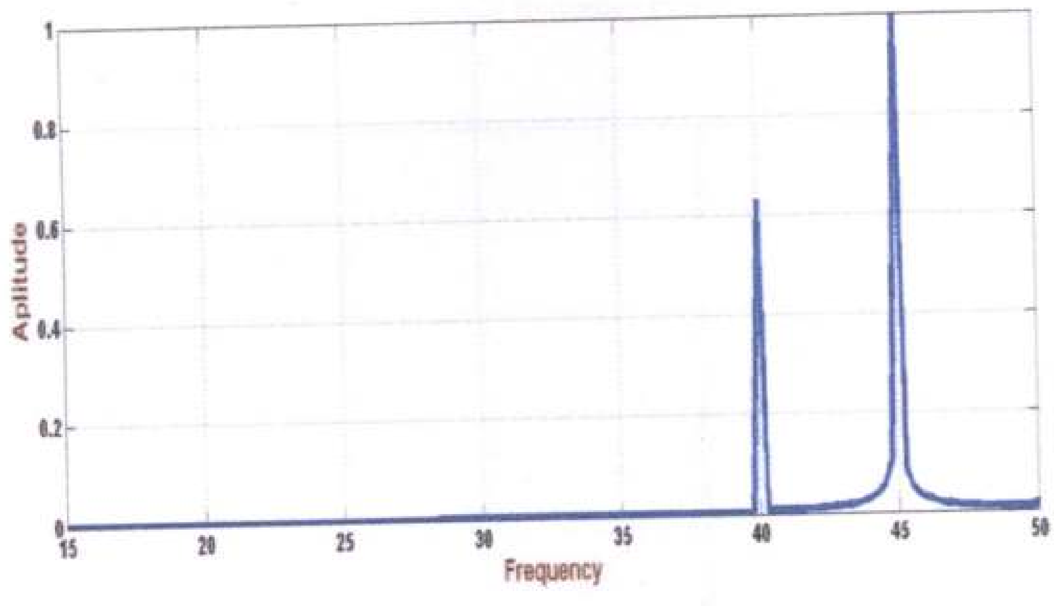

Fig 13.4 : Illustration of the mechanics of the injection locking phenomena with frequency less than the free-running frequency

Amplitude= 40 & Frequency= 40 Hz

Though we see the two spectrum very near (thereby introducing the pulling effects), but this is not sufficient to obtain the locking phenomena of the oscillator.

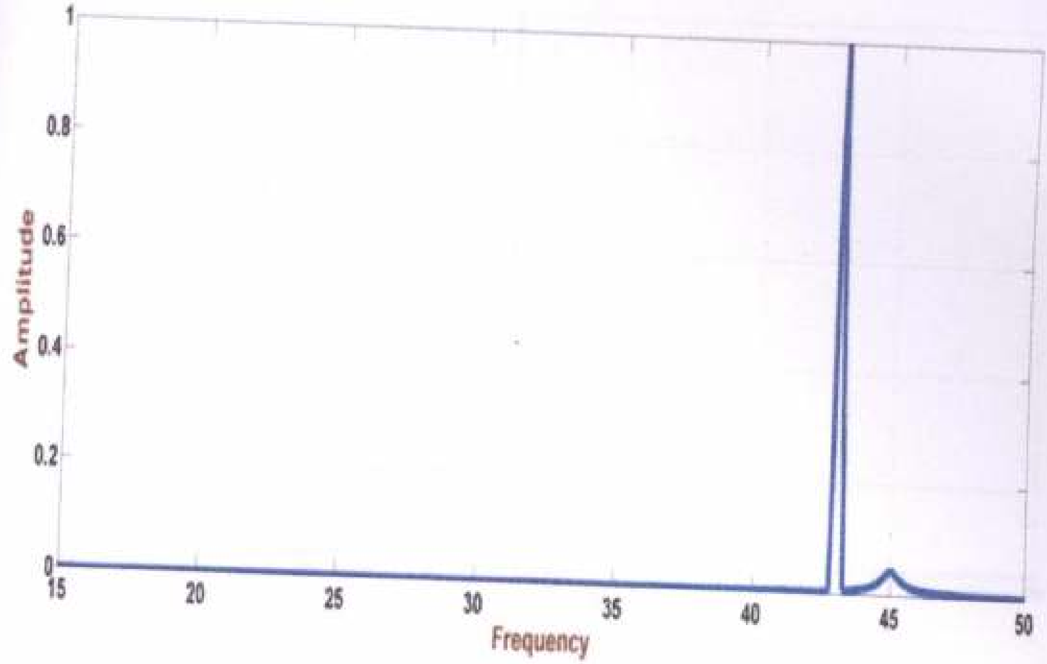

Fig 13.5 : Illustration of the mechanics of the injection locking phenomena with frequency approximately equal to the free-running frequency

Amplitude= 40 & Frequency= 43 Hz

The oscillator oscillates at the external frequency rather than the free-running frequency that means injection locking occurs.

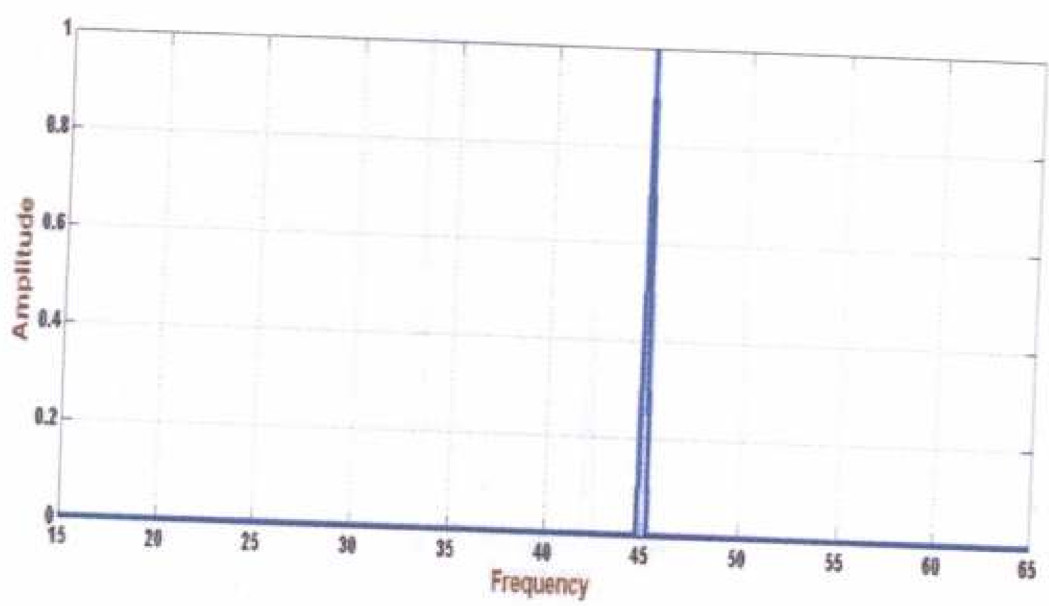

Fig 13.6 : Illustration of the mechanics of the injection locking phenomena at free-running frequency

Amplitude= 40 & Frequency= 45 Hz

Injection signal frequency matches with the free-running frequency.

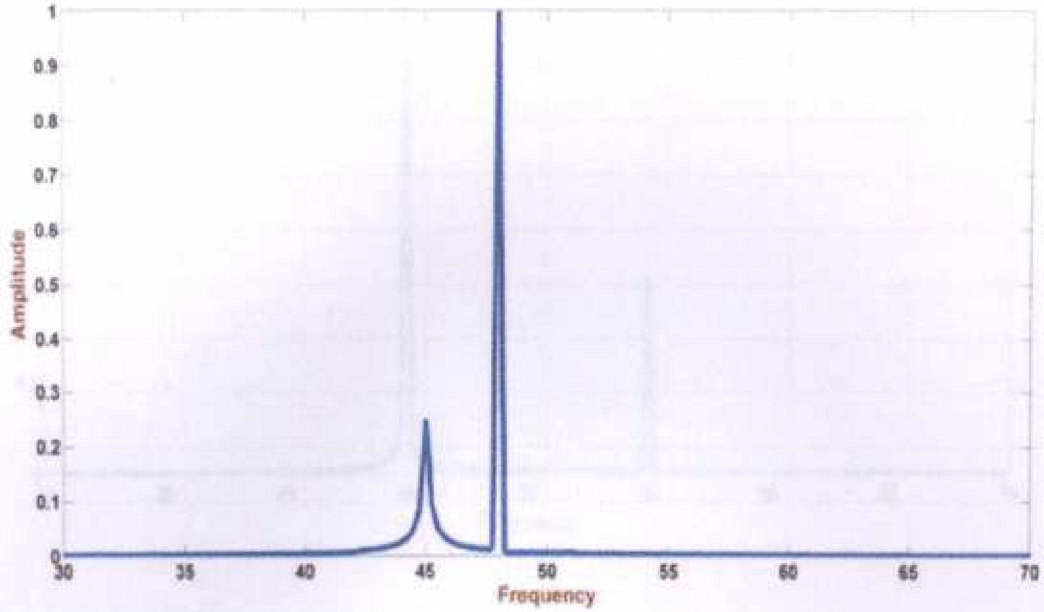

Fig 13.7: Illustration of the mechanics of the injection locking phenomena with frequency slightly greater than the injection frequency

Amplitude= 40 & Frequency= 48 Hz

As frequency increases beyond free-running frequency, spectrum for free-running frequency start to increase and spectrum for external frequency decreases. At 48 Hz locking phenomena still holds.

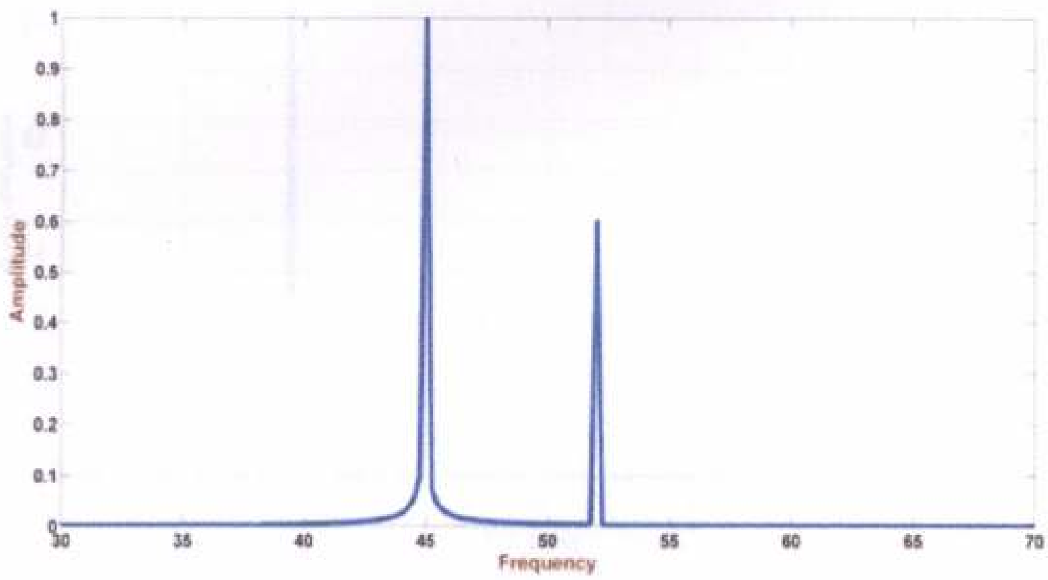

Fig 13.8: Illustration of the mechanics of the injection locking phenomena with slightly greater injection frequency

Amplitude= 40 & Frequency= 52 Hz

Spectrum for free-running frequency increases and spectrum for external frequency decreases.

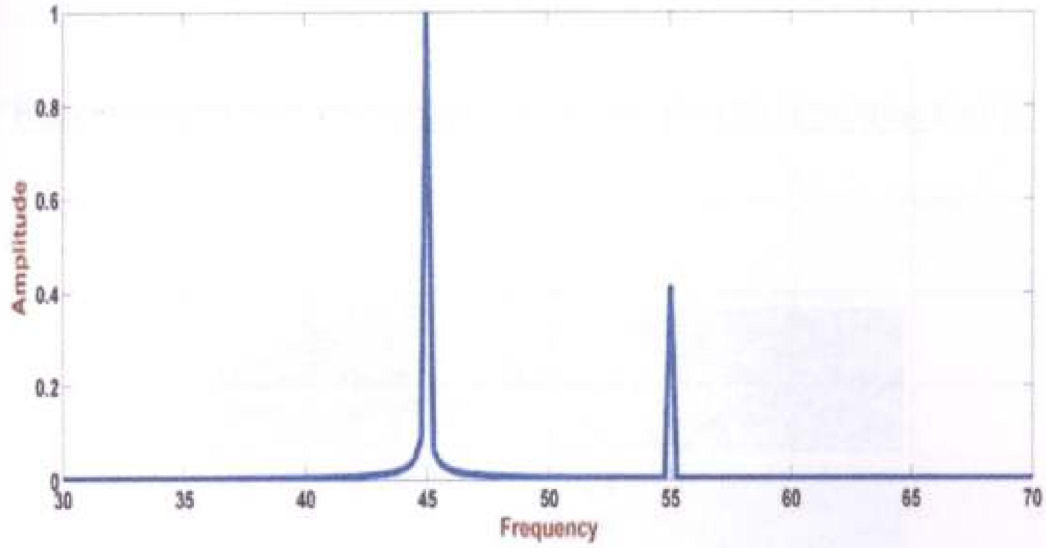

Fig 13.9: Illustration of the mechanics of the injection locking phenomena with injection frequency greater than the free-running frequency

Amplitude= 40 & Frequency= 55 Hz

Spectrum for free-running frequency increases and spectrum for external frequency decreases.

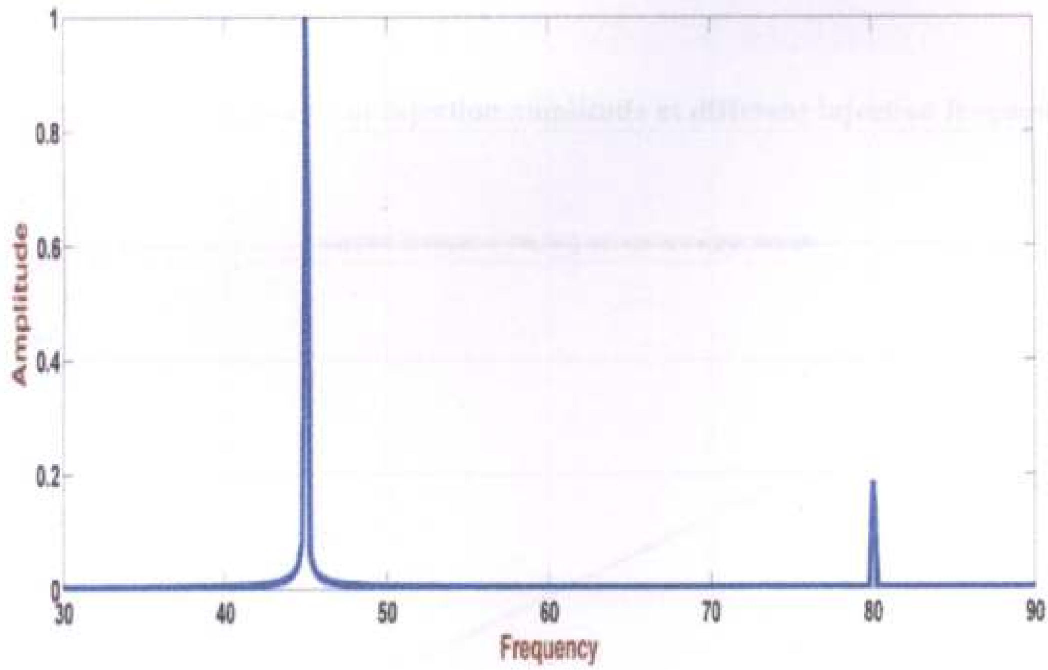

Fig 13.10: Illustration of the mechanics of the injection locking phenomena with slightly greater injection frequency

Amplitude= 40 & Frequency= 80 Hz

Spectrum for free-running frequency increases and spectrum for external frequency decreases and as we continuously increase the external signal frequency the oscillator will again oscillates at the free-running frequency rather than external frequency.

<u>Experimental data and graph for detuning with injection amplitude</u>

Frequency detuning in Hz	Injection Amplitude
1	25
3	73
5	117
7	165

Table 2:- simulated value of injection amplitude at different injection frequencies.

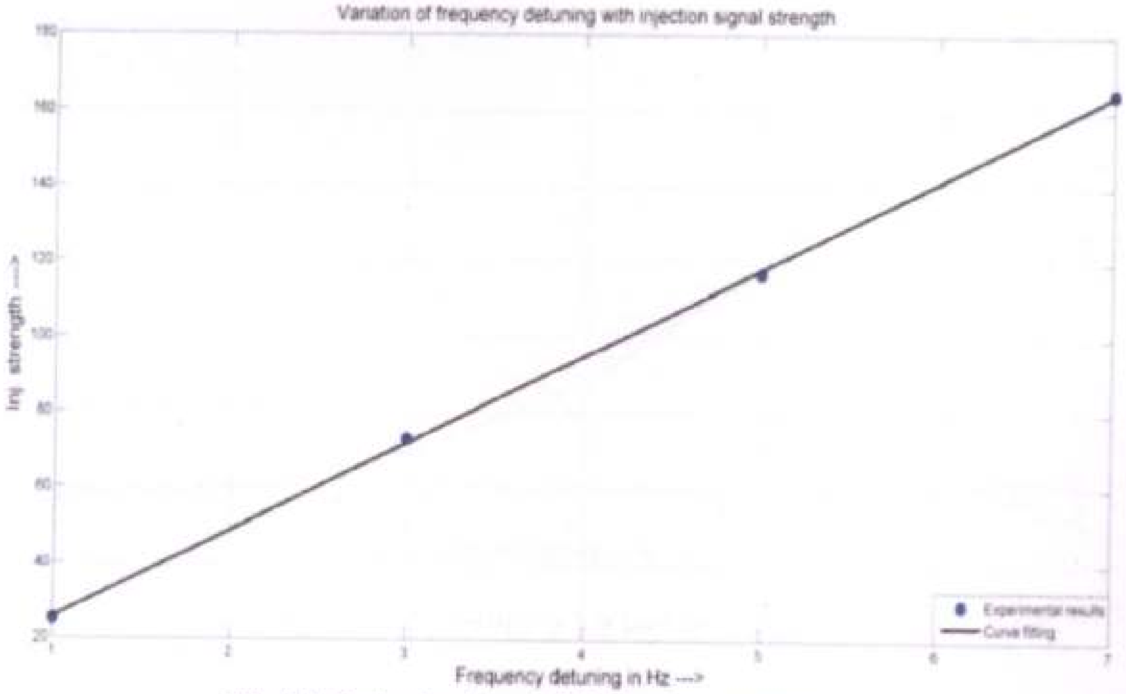

Fig 14: Detuning freq VS injection amplitude

Experimental data & graph for frequency detuning with locking range

Gain	Frequency Detuning	Locking Range
10	45-41=4	10
20	45-38=7	22
30	45-35=10	45
40	45-33=12	75
50	45-31=14	159
60	45-30=15	320
70	45-28=17	392

Table 3:- simulated values for locking range at different detuning frequencies

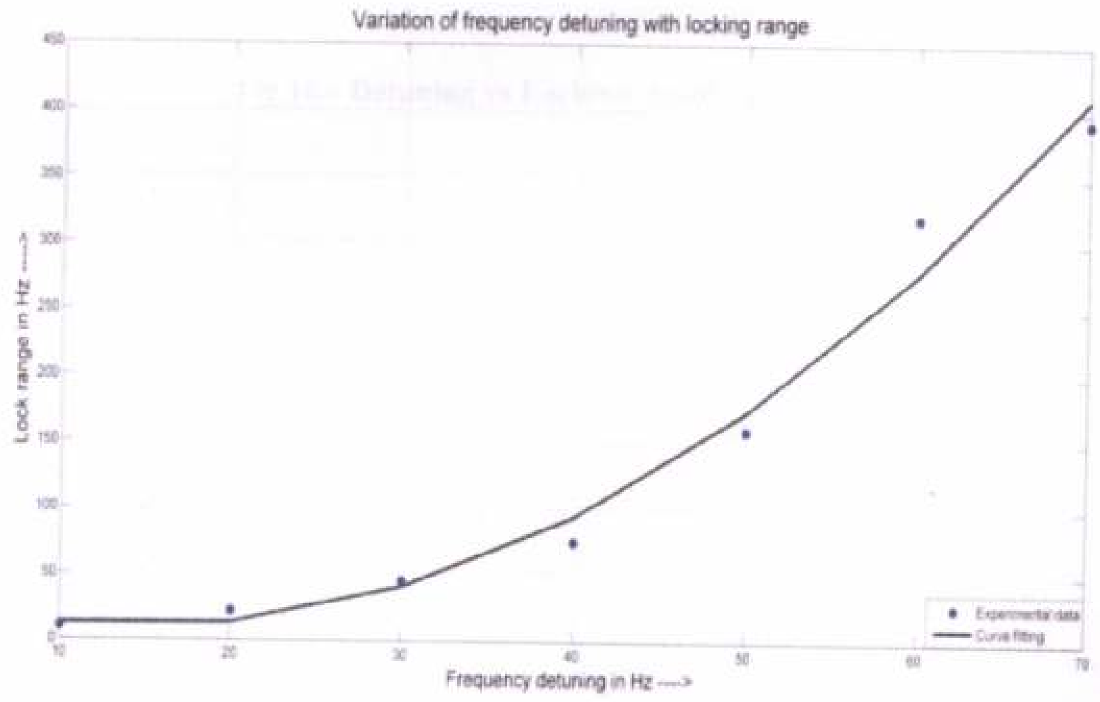

Fig 15: Freq detuning VS locking rang

Simulated Result from Tina- Ti software

The oscillator detuning frequency versus locking range graph is shown in Fig. 9. The free running frequency of the oscillator is 73 Hz. The experimental validation is done with the help of TINA-Ti.

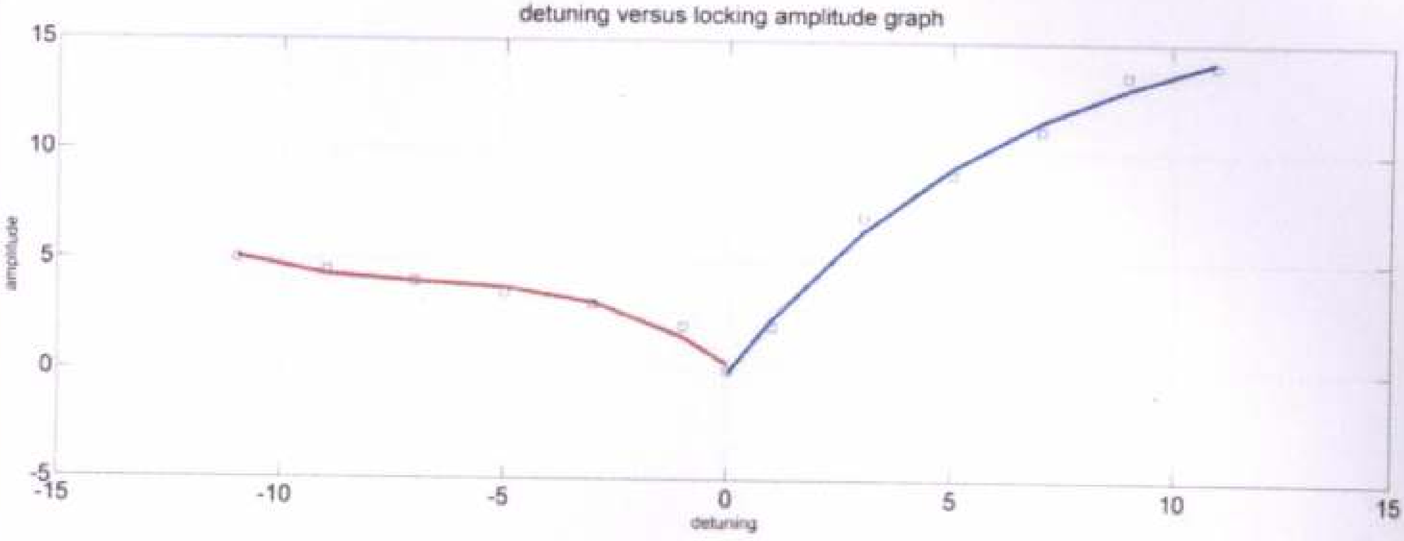

Fig 16:- Detuning vs Locking Amplitude Graph

Conclusion

Various a aspects of the injection locking phenomenon are studied and a locking equation is derived for the same. Lock range variations, detuning are studied and simulated using Matlab and Tina. The theoretical findings coupled with the experimental results (simulated) are presented in this project report.

Future works

A novel method for studying the injection synchronization of a two-port nonlinear oscillator with its parameter has been studied. Further to this work, a frequency modulated signal can be injected to the oscillator and FM to AM conversion by the injection locked oscillator can be observed. The figure of merit as a function of modulator index and other associated parameters can also be studied in future studies.

References

1. E. Normann, "The inductance-capacitance oscillator as a frequency divider," in *Proc. IRE*, vol. 24, Oct. 1946, pp. 799–803.
2. C. J. M.Verhoeven, "Ahigh-frequency electronically tunable quadrature oscillator," *IEEE J. Solid-State Circuits*, vol. 27, pp. 1097–1100, July 1992.
3. A. Rofougaran *et al.*, "A 900 MHz CMOS LC oscillator with quadrature outputs," in *IEEE ISSCC Dig. Tech. Papers*, Feb. 1996, pp. 392–393.
4. J. Kim and B. Kim, "A low phase noise CMOS LC oscillator with a ring structure," in *IEEE ISSCC Dig. Tech. Papers*, Feb. 2000, pp. 430–431.
5. B. Razavi, "A study of injection locking and pulling in oscillators," *IEEE J. Solid-State Circuits*, vol. 39, no. 9, pp. 1415–1424, Sept. 2004.
6. R. Adler,"A study of locking phenomena in oscillators," *Proc. IEEE*, vol. 61, pp. 1380–1385, Oct. 1973.